6.5 HW SOL

May 4, 2020 9:47 PM

Name:________________________________ Date:______________________________

6.5 Counting Using the Sticks and Stones Method

1. If I have 10 identical stones, how many ways can I split them into 4 groups?

   O_O_O_O_O_O_O_O_O_O

   $= {}_9C_3$

   - 9 spots in between
   - Choose 3 to split into 4 groups.

2. If "a", "b", and "c" are whole numbers, how many combinations of (a,b,c) are there such that:
   $a+b+c=9$

   ① whole numbers, so zero is allowed
   ② sum is 9 → 9 rocks
   ③ 3 variables → 2 sticks.
   11 spaces → ${}_{11}C_2$ //

3. How many ways can four whole numbers (a,b,c,d) add to 15?

   ① sum is 15 → 15 rocks
   ② 4 var. → 3 sticks
   ③ 18 spaces → ${}_{18}C_3$

4. Sandy wants to buy four donuts from Tim Hortons. She has 15 different flavours to choose from. How many different combinations of donuts can she buy her four donuts?

   _ 1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 _ 10 _ 11 _ 12 _ 13 _ 14 _ 15

   Fixed/.

   $(14+4)C_4 = {}_{18}C_4$ //

   - Put a stick left to a number means you choose that flavour
   - Consider only 14 rocks, where one rock must be on the far right

5. Tom walks to a buffet to pick his food. There are 5 types of meat, chicken, beef, pork, lamb, and fish. He can have up to 6 servings of any meat he wants. How many can he select his food?

   _ C _ B _ P _ L _ F

   ↓ Fixed.

   - 4 rocks
   - 6 servings

   $\Rightarrow {}_{10}C_4$

6. Suppose "a", "b", "c", and "d" are natural numbers, how many different combinations of (a,b,c,d) are there such that $a+b+c+d=15$? SAME AS Q3 //

   - sum is 15 → 15 rocks
   - 4 variables → 3 sticks
   - ${}_{18}C_3$ //

7. In a basketball game, in a team of 12 players, six of the players took 45 shots altogether. How many ways can the 45 shots be distributed to the 6 players?

   (i)

   $a+b+c+d=45$

   Rocks $=45$ sticks $=3$.

   ${}_{48}C_3$ //

8. You are ordering a dozen doughnuts (12) and need to choose from among four flavors: glazed, powdered, cream-filled, and jelly-filled. How many different doughnut orders are possible if you must choose atleast one of each flavour?

(1) $a+b+c+d=12$

(2) $a,b,c,d \geq 1$

So CANNOT BE ZERO!

$(8+3)C_3$

1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 _ 10 _ 11 _ 12

11 spaces, choose 3.

$11C_3$

9. David has 25 coins in his pocket: nickels, dimes, pennies, and quarters. How many different combinations of coins can he have?

N D P Q

(1) $N+D+P+Q=25$

(2) 25 Rows 3 sticks

(3) $28C_3$

10. Each of the numbers from 1 to 10 are placed in a bag and three numbers are taken out with replacement. ~~How many ways~~ can the three numbers drawn add to a sum of 20?

1, 9, 10 | 2 10 8 | 4 10 6
3 10 7 | 4 9 7
3 9 8 | 5 9 6
5 8 7

~~6 10 4~~ ~~6 9 5~~ ~~6 8 6~~ 7 ~~10 3~~ ~~7 9 4~~ ~~7 8 5~~ 7 ~~7 6~~

8 ways (NO SMART WAYS)

11. Suppose Tom goes to a different buffet to eat. If there are "X" types of meat to choose from and he can have up to "Y" servings of any meat he wants, how many can he select his food in terms of "x" and "Y"?

ie suppose $x=5$, $y=2$.
THEN
0,1,2 0,1,2 0,1,2 0,1,2 0,1,2
(3) · (3) · (3) · (3) · (3)
$= 3^5$

$\therefore \boxed{(y+1)^x}$

12. Liz went to a buffet where they served 5 types of dessert. For the 1st type of dessert you are only allowed 1 serving. For the 2nd type of dessert you are allowed up to 2 servings. For the 3rd type of dessert, you are allowed up to 3 servings. Same pattern applies to the fourth and fifth type of dessert. If Liz is to have 10 servings altogether, how many ways can he order her desserts?

a = 0, 1
b = 0, 1, 2
c = 0, 1, 2, 3
d = 0, 1, 2, 3, 4
e = 0, 1, 2, 3, 4, 5

a _ | b _ _ | c _ _ _ | d _ _ _ _ | e _ _ _ _ _ |

$\boxed{15C_5} =$

13. Let a,b,c be three different whole numbers. What is the number of triple pairs (a,b,c) such that the equation is true? $a+b+c \leq 18$

(1) WHOLE NUMBERS → VALUE CAN BE ZERO

(2) less than or equal to 18 → last stick can be anywhere

18 ROWS, 3 STICKS

$21C_3$.

14. Given that $0 \le x_1 \le x_2 \le x_3 \le x_4 \le 6$, how many ways are there to select (x_1, x_2, x_3, x_4) ?

_ 0 _ 1 _ 2 _ 3 _ 4 _ 5 _ 6

① put any stick(s) left to that number means you select that value.

② last number is fixed

fixed

6 rows 4 sticks → $_{10}C_4$ //

15. Four standard six sided dice are rolled. How many ways can you get a sum of 12?

① $a+b+c+d = 12$.

② a, b, c, d b/w 1 to 6 (not zero)

$12-4=8$ 8 rows.

3 sticks

$_{11}C_3 = 165$.

③ Take out bad combos:

9, 2, 1 → 6
8, 2, 2 → 3
8, 3, 1 → 6
7 4 1 → 6
7 2 3 → 6

27.

Total = 165 − 27 = 138

16. Given that $0 \le x_1 \le x_2 \le x_3 .... \le x_6 \le 20$, where $x_1, x_2, ...$ *and* x_6 are whole numbers. How many different combinations of $(x_1, x_2, x_3, x_4, x_5, x_6)$ can there be?

_ 0 _ 1 _ 2 _ 3 _ 4 _ 5 _ 6 . . . . _ 18 _ 19 _ 20

① 20 rows, last number on the right is fixed [0-19]

② 6 sticks, place your number to the left of each # to choose that number.

$_{26}C_6$

17. Suppose we have 5 integers that add up to 25. The first number must be greater or equal to 1. The 2nd number must be greater or equal to 2. The third number must be greater or equal to 3, fourth number greater or equal to 4, and the fifth number greater or equal to 5. How many combinations are there for the five numbers? $(x_1, x_2, x_3, x_4, x_5)$

$a+b+c+d+e = 25$
$-1 \quad -2 \quad -3 \quad -4 \quad -5 \quad -15$

∴ $a+b+c+d+e = 10$.

∴ sum to 10 → 10 rows

4 bars → 4 sticks

∴ $_{14}C_4$

fixed

x_1 | x_2 || x_3 ||| x_4 |||| x_5 |||||

after set.

25 sticks → 15 sticks

10 sticks left

$\boxed{_{14}C_4}$ ✓

$a+b+c = 10$

$a \ge 1$ $b \ge 2$ $c \ge 3$

fixed

4 sticks left

$_6C_2 = 15$ → 15

1 2 8	2 2 6	3 2 5	4 2 4	5 2 3
1 3 7	2 3 5	3 3 4	4 3 3	①
1 4 6	2 4 4	3 4 3	②	
1 5 4	2 5 3	③		
1 6 3	④			
⑤				

18. The polynomial $(a+b+c+d)^{15}$ is expanded. How many terms in the expansion contains all four variables "a", "b", "c", and "d"?

① $(a+b)^2 = a^2 + ab + b^2$

$(a+b)^3 = a^3 + a^2b + ab^2 + b^3$.

② Any term with all variables must have exponent greater than 1.

③ All terms with all 4 variables are in the form of $a^x \cdot b^y \cdot c^z \cdot d^w$.

So $x+y+z+w = 15$ $(x,y,z,w) \geq 1$

$15-4 = 11$ rocks, 3 sticks → $_{14}C_3$.

19. Suppose "a" , "b", "c", and "d" are all positive odd integers. How many different combinations of (a,b,c,d) are there such that a+b+c+d = 98

① Let $a = 2m+1$, $b = 2n+1$, $c = 2p+1$, $d = 2q+1$ where m, n, p, q are whole numbers.

② $2m+1+2n+1+2p+1+2q+1 = 98$

$2m+2n+2p+2q = 94$

$m+n+p+q = 47$

Sum is 47 → 47 rocks

4 var → 3 sticks.

$_{50}C_3$.

20. There are nine chairs in a row and 6 chairs are to be seated by students. 3 Professors arrive at the chairs before any students show up. Professors "X", Y", and "Z" decides to sit down before any students arrive. How many ways can the professors choose their seats if each professor must be in between two students? AHSME 1994

① These 3 spots are for the 3 professors $= 3!$ (6 ways)

X Y Z

② 6 students, 4 student spaces are left.

s [ ] s [ ] s [ ] s

③ _ [ ] _ [ ] _ [ ] _ → 5

2 student spaces left

$_5C_2 = \frac{5 \times 4}{1 \times 2} = 10$

④ Total $= 6 \times 10 = 60$ ways.

21. A giant spider has eight legs and needs to wear 8 socks and 8 shoes. The spider needs to wear a sock before it puts a shoe on any of its legs. How many different orders can the spider put on all 8 socks and shoes.

Letter the legs from A, B, C, D, E, F & H.

Since each leg will wear 2 things, then each leg will need 2 letters.

So we have AA, BB, CC, DD, EE, FF, GG, HH.

Number of ways to arrange these letters:

$\frac{16!}{(2)^8}$.

Since sock is always worn before shoe, then

1st A → sock

2nd A → shoe.

When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as

$$\frac{n}{6^7},$$

where n is a positive integer. What is n?

(A) 42 **(B)** 49 **(C)** 56 **(D)** 63 **(E)** 84